Gravity current into an ambient fluid with an open surface, , and application to shallow-water model

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Consider the idealized steady-state gravity current of height h and density ρ_1 that propagates into an ambient motionless fluid of height H and density ρ_2 with an upper surface open to the atmosphere (open channel) at high Reynolds number. The current propagates with speed U and causes a depth decrease χ of the top surface. This is a significant extension of Benjamin's (1968) seminal solution for the fixed-top channel $\chi = 0$. Here the determination of χ is a part of the problem. The dimensionless parameters of the problem are $a = h/H$ and $r = \rho_2/\rho_1$. We show that a control-volume analysis determines $\tilde{\chi} = \chi/H$ and $Fr = U/(g'h)^{1/2}$ as functions of a, r, where $g' = (r^{-1} - 1)g$ is the reduced gravity. The system satisfies balance of volume and momentum (explicitly), and vorticity (implicitly). We present solutions for two "models" concerning the profile of the flow above the current: vortex-sheet, and vortex-wake. The predicted flows are in general dissipative, and thus physically-energetically valid only for $a \leqslant a_{\max}(r) \approx 0.5r$ where non-negative dissipation appears. The open-surface $Fr(a, r)$ is smaller than Benjamin's $Fr_b(a)$, but the reduction is not dramatic, typically a few percent. For a Boussinesq system with $r \approx 1$, we obtain $\tilde{\chi} \ll 1$, and the present Fr, and dissipation results differ only slightly from Benjamin's classical predictions, as expected. In general gravity currents are time-dependent and amenable to description by two-layer shallow-water (SW) equations. We show that the Fr and $\tilde{\chi}$ solution are useful "jump" conditions for the SW predictions.