Investigation of the near-wake square cylinder by means of the Empirical Mode Decomposition

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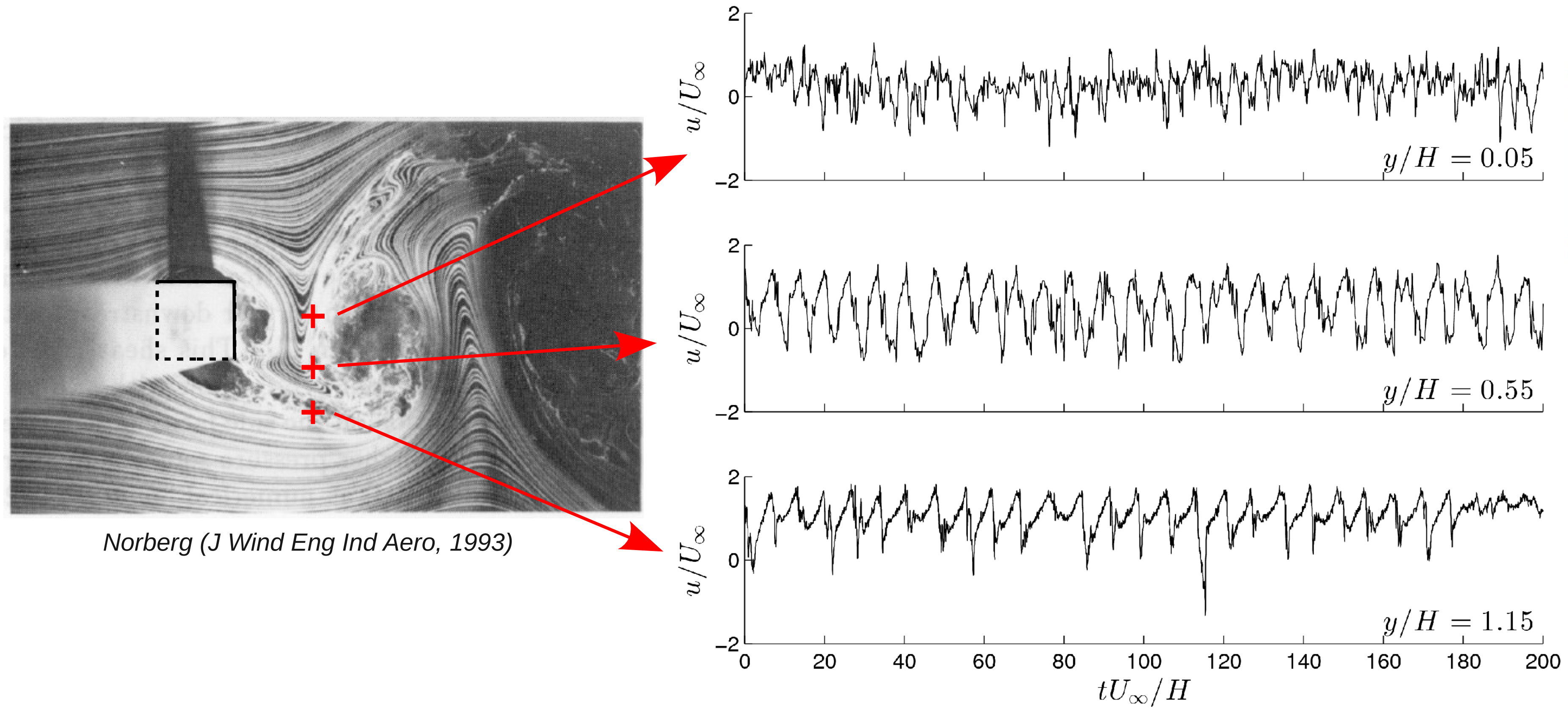
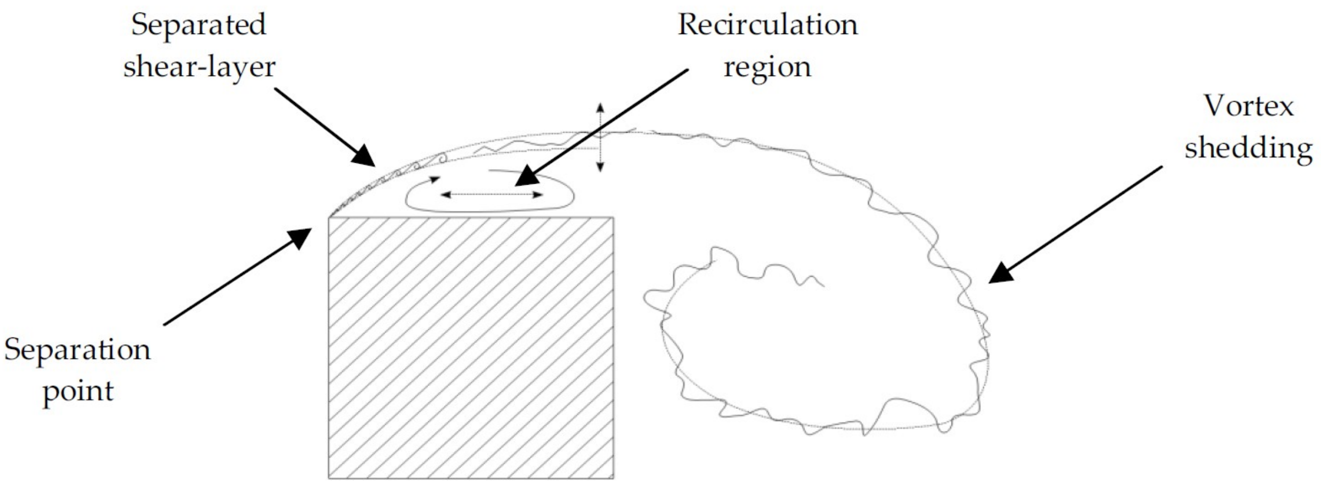


Norberg (J Wind Eng Ind Aero, 1993)

→ Co-existence of both coherent ("quasi-periodic") and incoherent ("random") motions

Hussain (Phys Fluids, 1983)

A) Context of the work: bluff-body wake



Two main issues:

→ Turbulence modelling — Separation features (length, position …) are badly predicted by URANS (Unsteady Reynolds-Averaged Simulation)

→ Flow control design — Relevant Reduced-Order Models

→ **Deep understanding of the physical mechanisms governing turbulent separated flows**

A) Context of the work: coherent vs turbulent

Triple decomposition *Hussain and Reynolds (J Fluid Mech, 1970)*

$$u(\vec{x},t) = U(\vec{x}) + \tilde{u}(\vec{x},t) + u'(\vec{x},t)$$

Turbulent fluctuation

Time
averaged

Coherent fluctuation

Interplay?

Coherent structure eduction:

- ➡ Phase Averaging (define phase ⇒ only periodic)
  (Lyn and Rodi, 1994)
- ➡ Proper-Orthogonal Decomposition (not relevant for dynamics)
  (Lumley, 1967; Berkooz et al., 1993)
- ➡ Dynamical Mode Decomposition (non linear systems, frequency selection)
  (Schmid, 2010)
  Koopman modes
  (Rowley et al., 2009)
- ➡ Wavelet transform (a priori basis of functions)
  (Farge et al., 2001)

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B) The Empirical Mode Decomposition: basics

The **Empirical Mode Decomposition** (or more generally the Huang-Hilbert Transform)

Huang et al. (Proc. R. Soc. Lond. A, 1998)

« detail » (high frequency)

$$s(t) = d(t) + r(t) \qquad \text{with} \qquad r(t) = \frac{e_{min}(t) + e_{max}(t)}{2}$$

« trend » (low frequency)

http://perso.ens-lyon.fr/patrick.flandrin/

$$\boxed{s(t) = \sum_{k=1}^{N} \mathrm{IMF}_k(t) + r_N(t)}$$

- ✔ Data-driven technique
- ✔ Instantaneous frequency

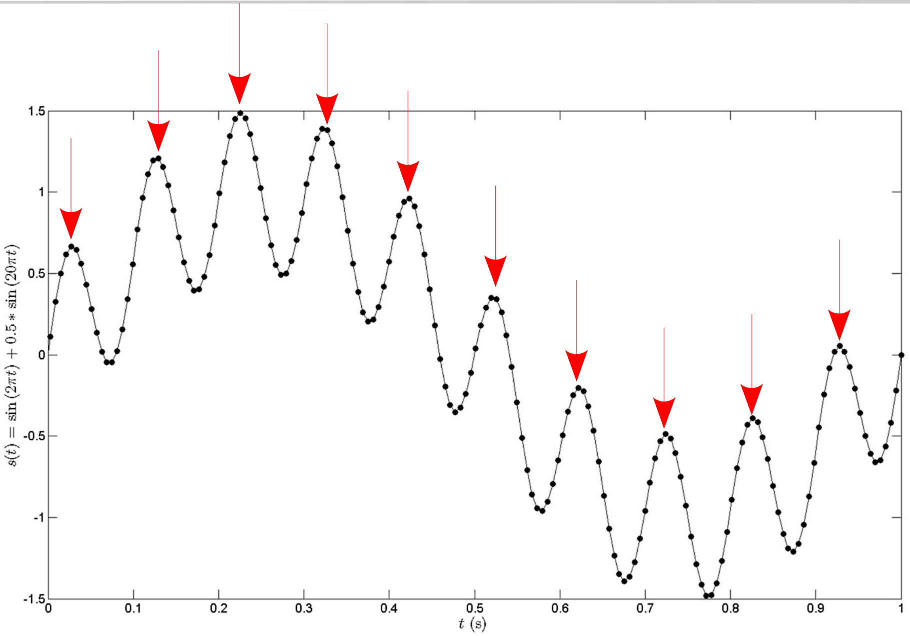
→ Non-linear and non-stationnary physics

B) The Empirical Mode Decomposition: example

$$s(t) = \sin 2\pi t + \frac{1}{2}\sin 20\pi t$$

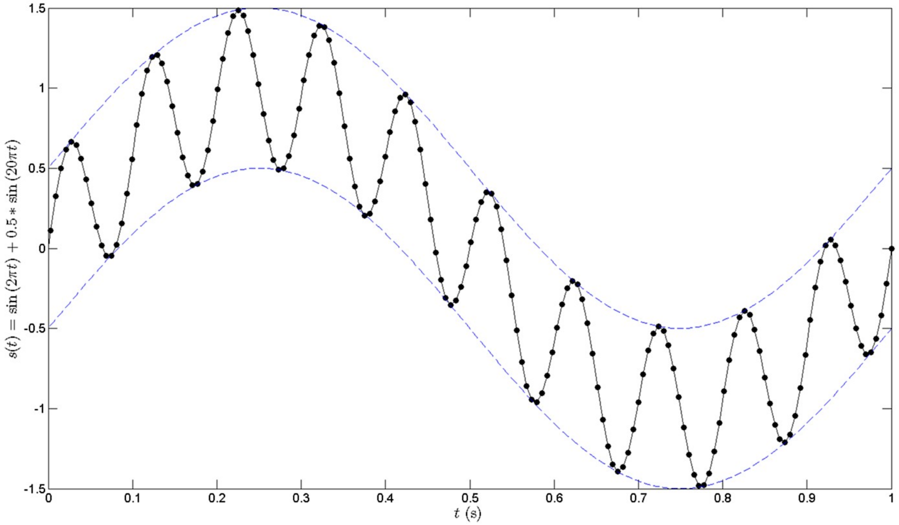
Rilling and Flandrin (2008)

- Local extrema detection
- Envelopes
- Local mean *r*(*t*)



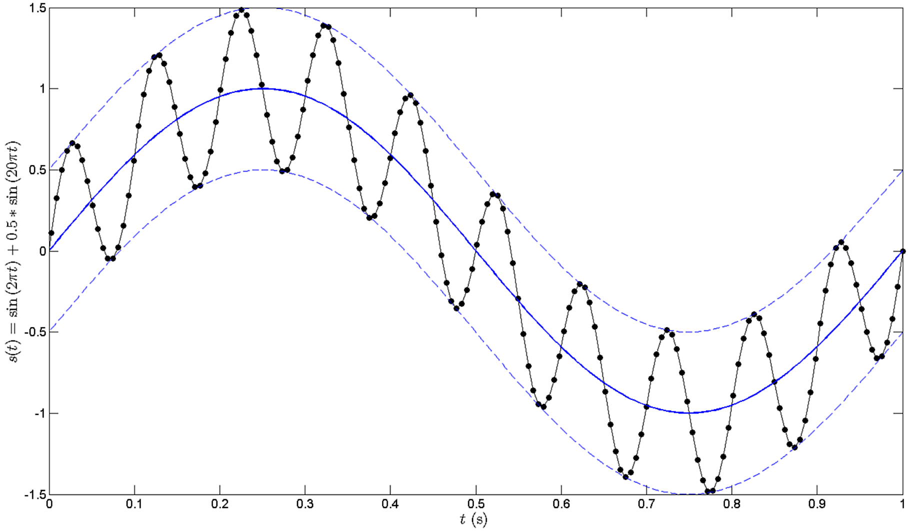
B) The Empirical Mode Decomposition: example

- Local extrema detection
- Envelopes
- Local mean *r*(*t*)



B) The Empirical Mode Decomposition: example

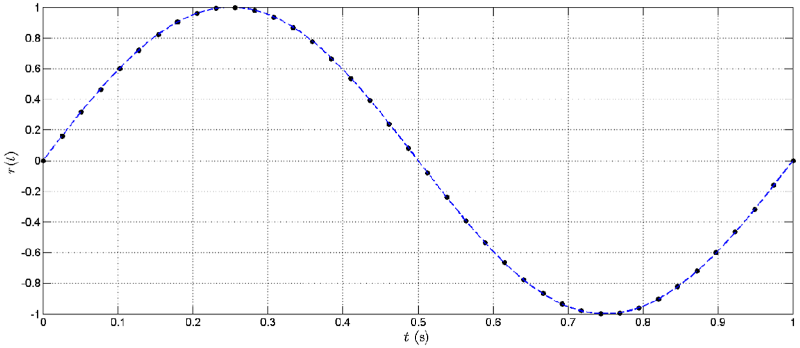
$$s(t) = \sin 2\pi t + \frac{1}{2}\sin 20\pi t$$

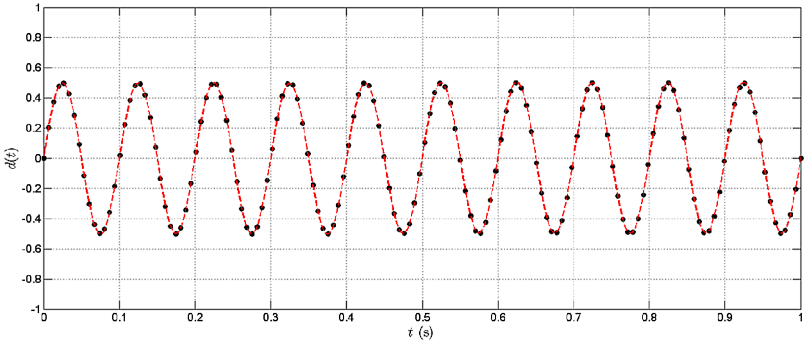


- ➢ Local extrema detection
- ➢ Envelopes
- ➢ Local mean *r*(*t*)

Trend: $\sin 2\pi t$

Detail: $\frac{1}{2}\sin 20\pi t$





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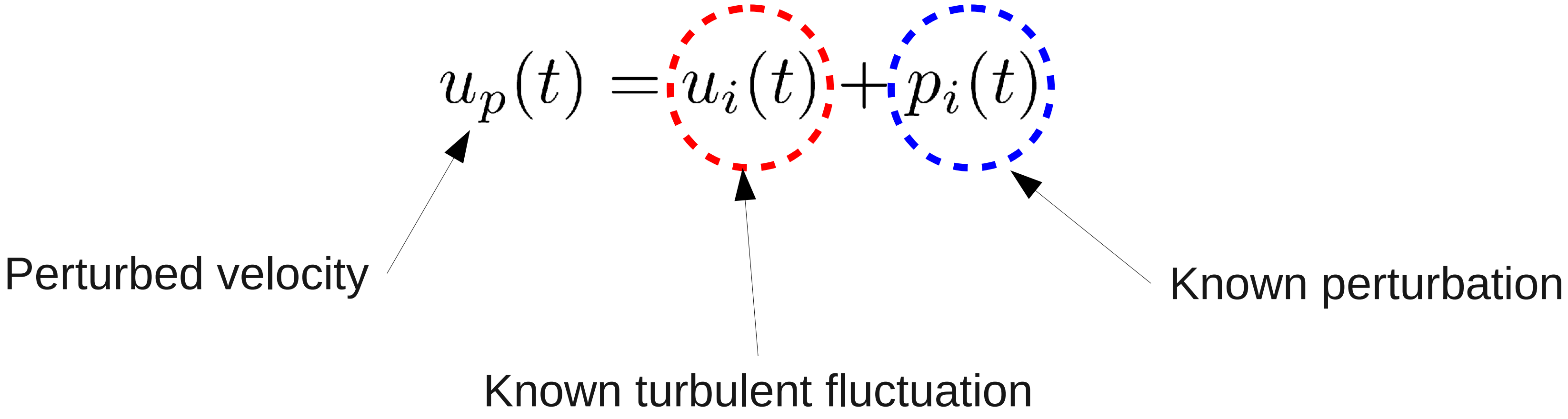
C) Performances of EMD: strategy

Assessment of EMD performances in turbulence?



$$u_p(t) = u_i(t) + p_i(t)$$

Perturbed velocity

Known turbulent fluctuation

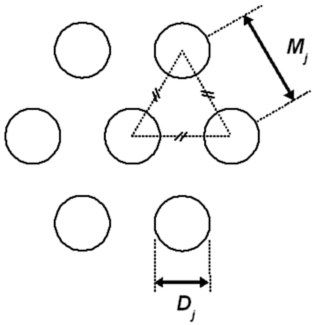
Known perturbation

Are we able to recover $u_i(t)$ and $p_i(t)$ from $u_p(t)$ by means of the EMD ???

C) Performances of EMD: strategy

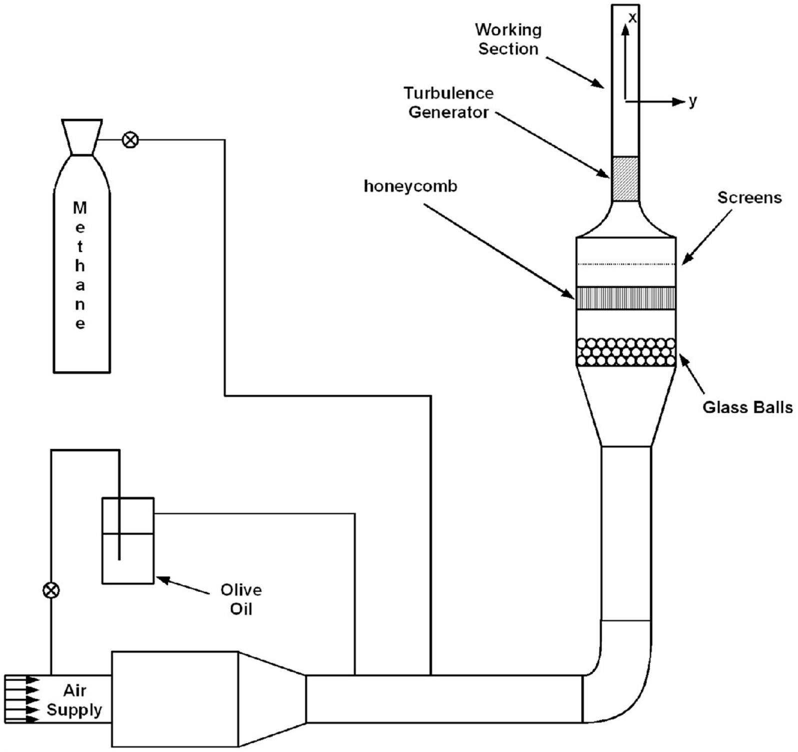
$$u_p(t) = u_i(t) + p_i(t)$$

Homogeneous and isotropic turbulence
Grid turbulence
Mazellier et al. (J. Turbulence, 2010)

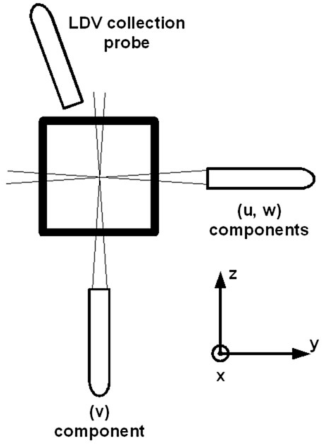


Perforated plate

D = 15 mm
M = 24 mm
σ = 67%



Experimental set-up

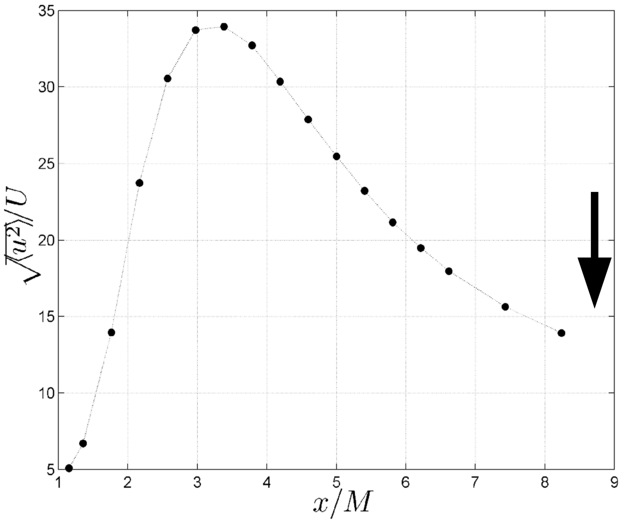


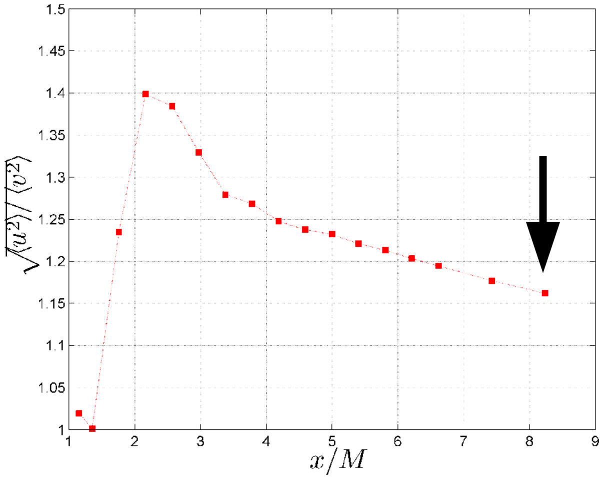
Seeding : olive oil (1µm, St<0.1)
ADL 3C : forward scattering(25°)
resolution ≈ η

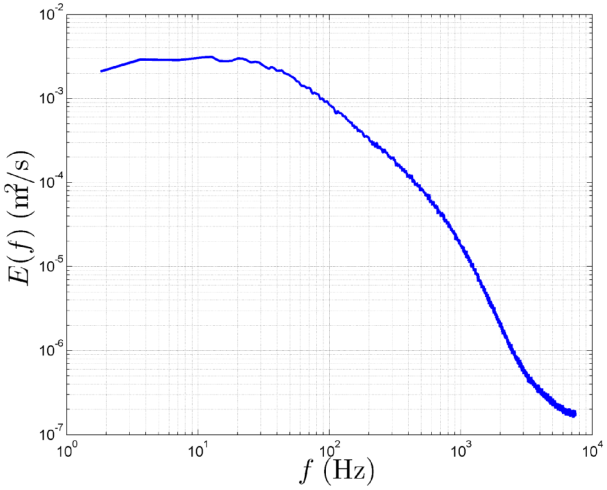
C) Performances of EMD: strategy

$$u_p(t) = u_i(t) + p_i(t)$$

Homogeneous and isotropic turbulence
Grid turbulence
Mazellier et al. (J. Turbulence, 2010)



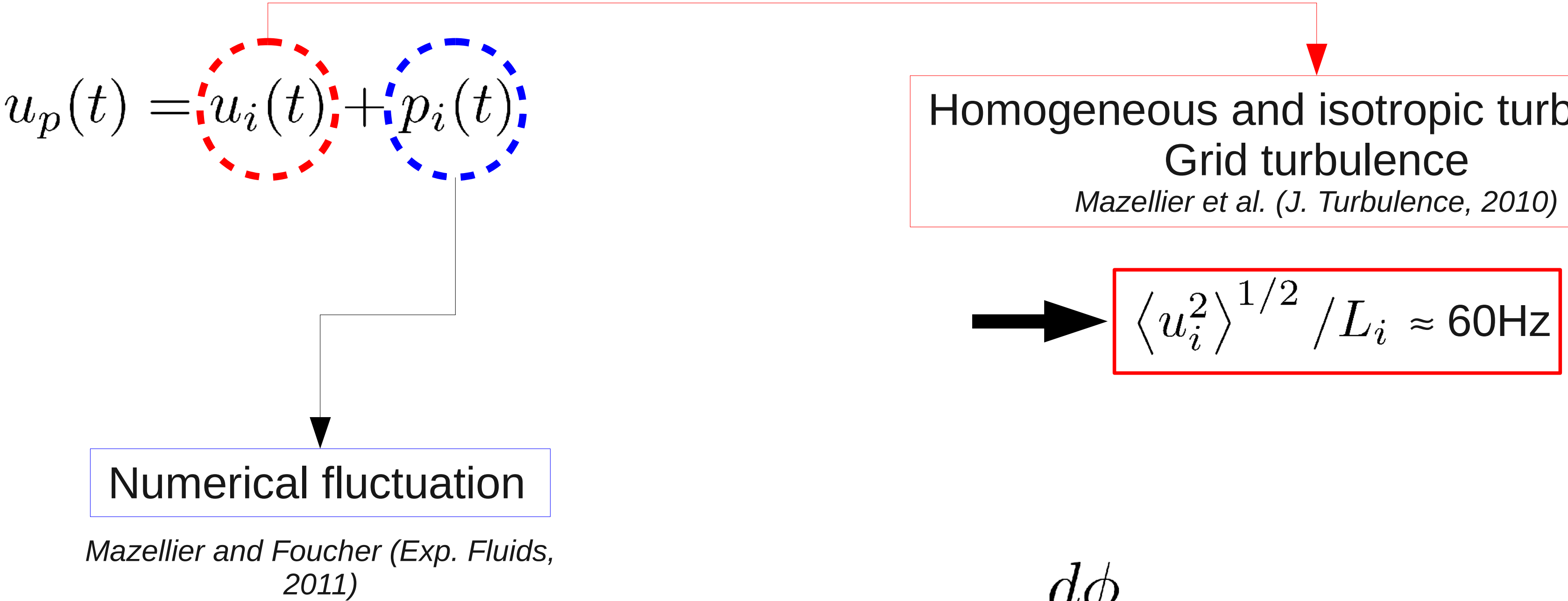




U^a (m/s)	$\langle u_i^2\rangle^{1/2}$ (m/s)	L_i^b (mm)	λ_i^c (mm)	η_i^d	Re_λ^e
4.1	0.57	9.4	1.6	0.1	59

[a] Streamwise mean velocity,
[b] Integral length-scale computed from Eq. (14),
[c] Taylor microscale computed from Eq. (15),
[d] Kolmogorov scale computed from Eq. (16),
[e] Taylor-based Reynolds number ($\equiv \langle u_i^2\rangle^{1/2}\lambda_i/\nu$ with ν the kinematic viscosity).

C) Performances of EMD: strategy



$$u_p(t) = u_i(t) + p_i(t)$$

Homogeneous and isotropic turbulence
Grid turbulence
Mazellier et al. (J. Turbulence, 2010)

$$\langle u_i^2 \rangle^{1/2} / L_i \approx 60\text{Hz}$$

Numerical fluctuation

Mazellier and Foucher (Exp. Fluids, 2011)

$$\frac{d\phi}{dt} = 2\pi f_p(t)$$

$$p_i(t) = a_p(t)\sin(\phi(t))$$

Low frequency flapping

Single tone

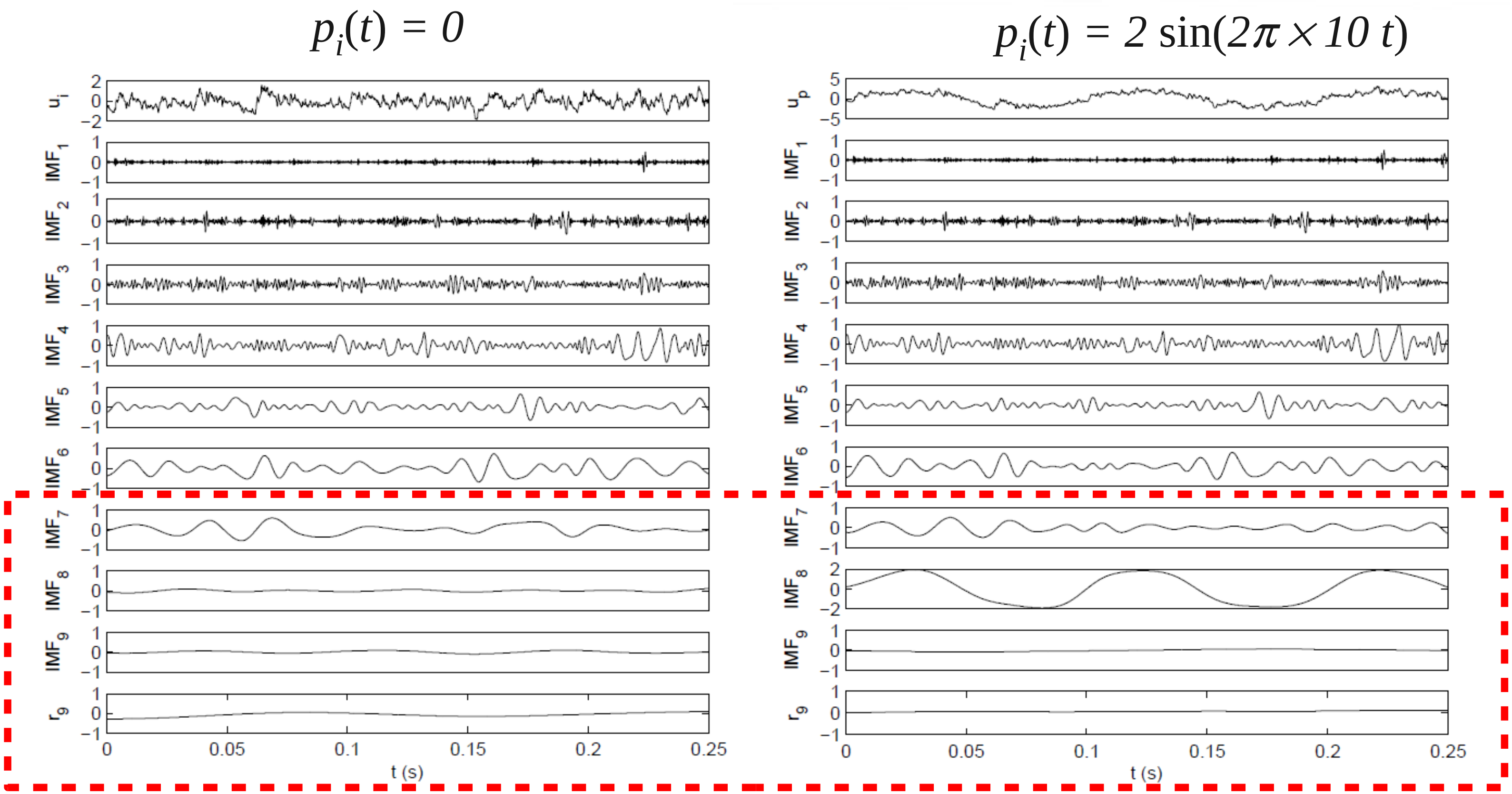
or

Non-stationary

$$a_p < 4\ \text{m/s}$$

$$f_p < 10\ \text{Hz}$$

C) Performances of EMD: example



$p_i(t) = 0$

$p_i(t) = 2\sin(2\pi \times 10\, t)$

First modes contain the high frequencies, while

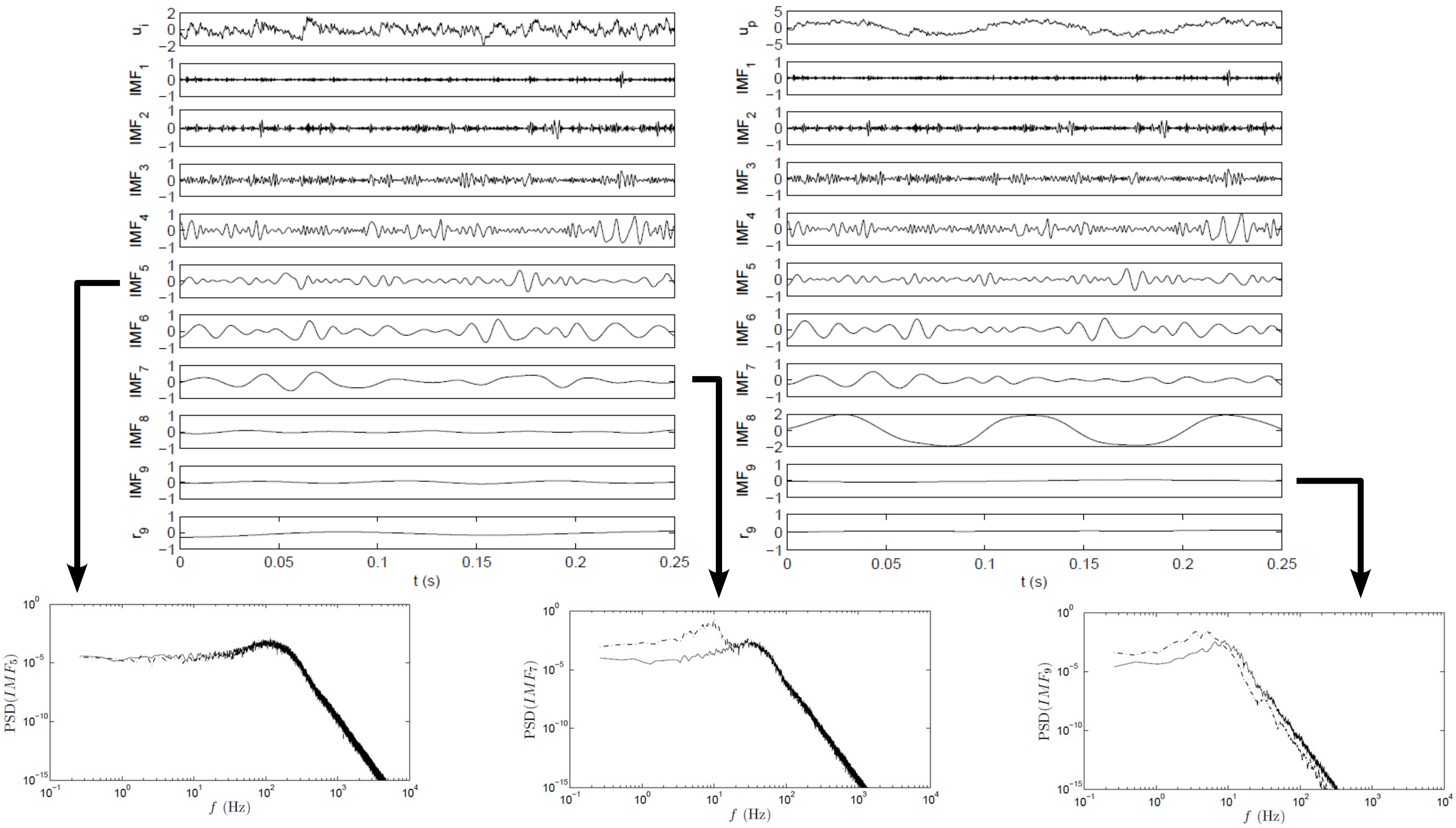
The perturbation is essentially concentrated on the highest order modes

→ One has to define a reliable criterion in order to discriminate between polluted and non polluted modes *Rilling and Flandrin (2008), Foucher and Ravier (Exp. Fluids, 2010)*

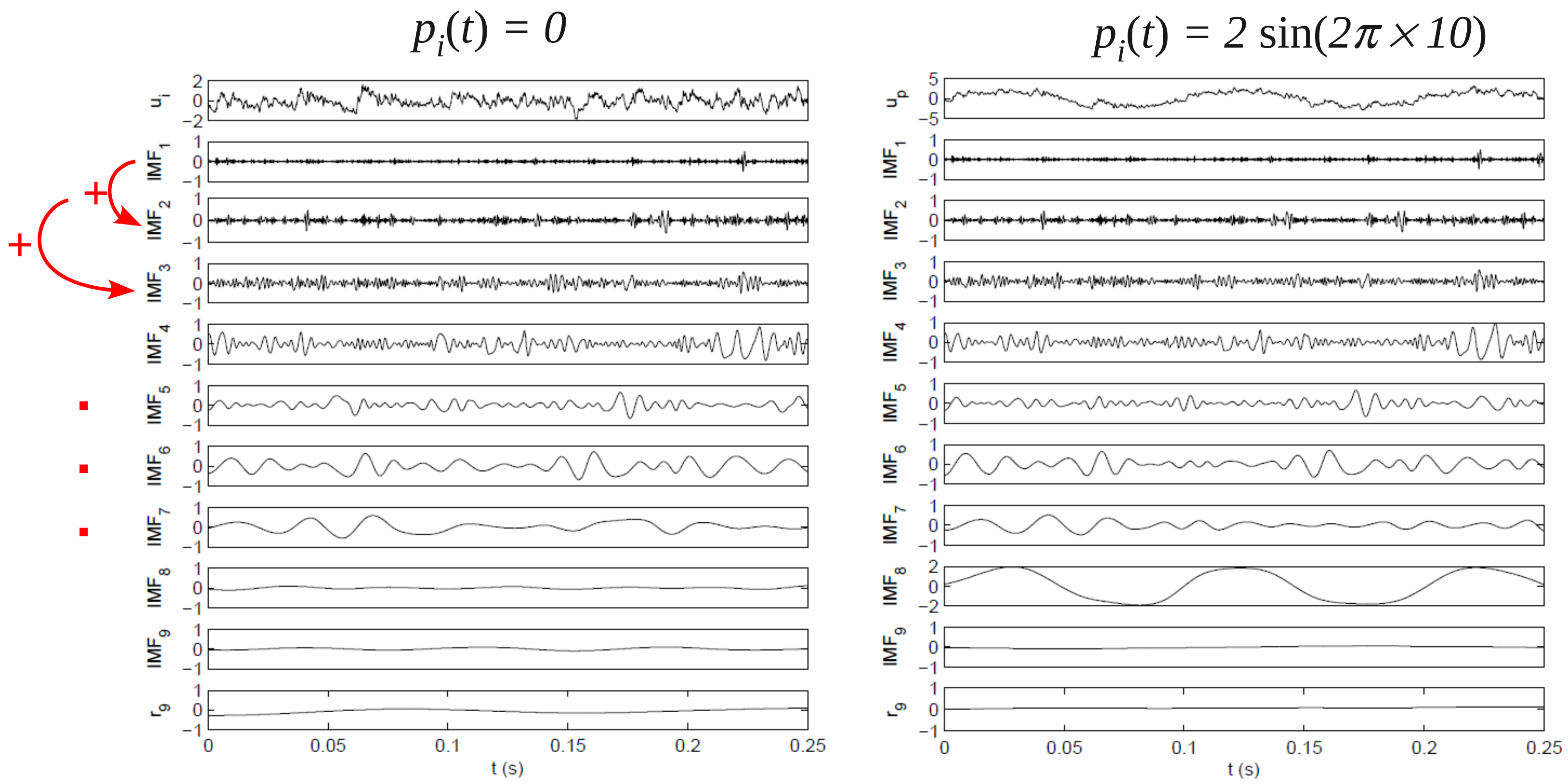
C) Performances of EMD: example

$p_i(t) = 0$

$p_i(t) = 2\sin(2\pi \times 10\, t)$



C) Performances of EMD: separation criterion



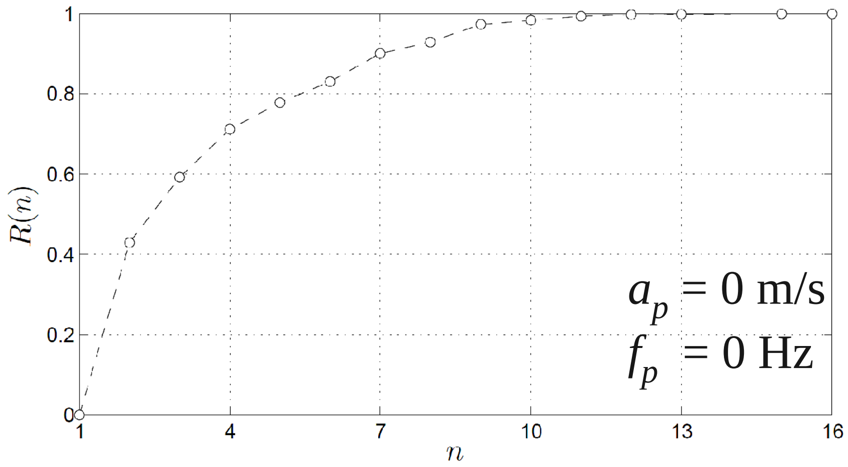
Step by step signal reconstruction from high to low frequencies

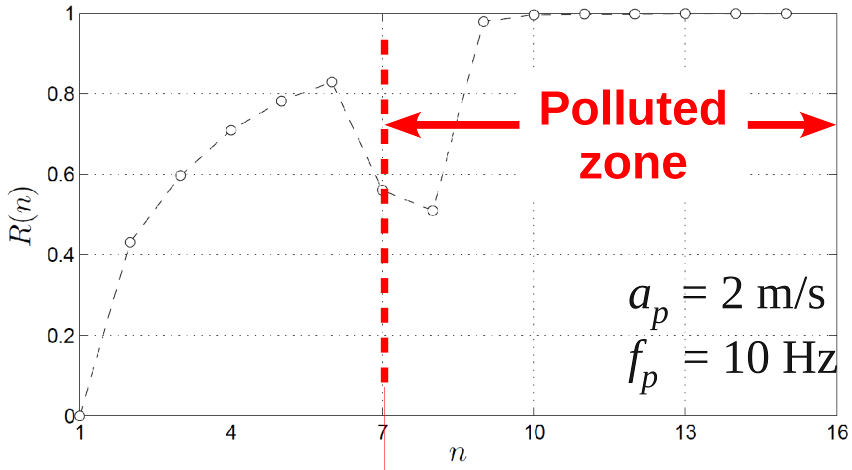
$$\begin{cases} u_n(t) = \sum_{k=1}^{n} \mathrm{IMF}_k(t) & \text{if } n < N, \\ u_n(t) = \sum_{k=1}^{n} \mathrm{IMF}_k(t) + r_N & \text{if } n = N, \end{cases}$$

C) Performances of EMD: separation criterion

« resemblance » criterion

$$\begin{cases} R(n) = 0 & \text{if } n = 1, \\ R(n) = \dfrac{\langle u_n(t)u_{n-1}(t)\rangle}{\sqrt{\langle u_n(t)^2\rangle}\sqrt{\langle u_{n-1}(t)^2\rangle}} & \text{if } 1 < n \le N \end{cases}$$





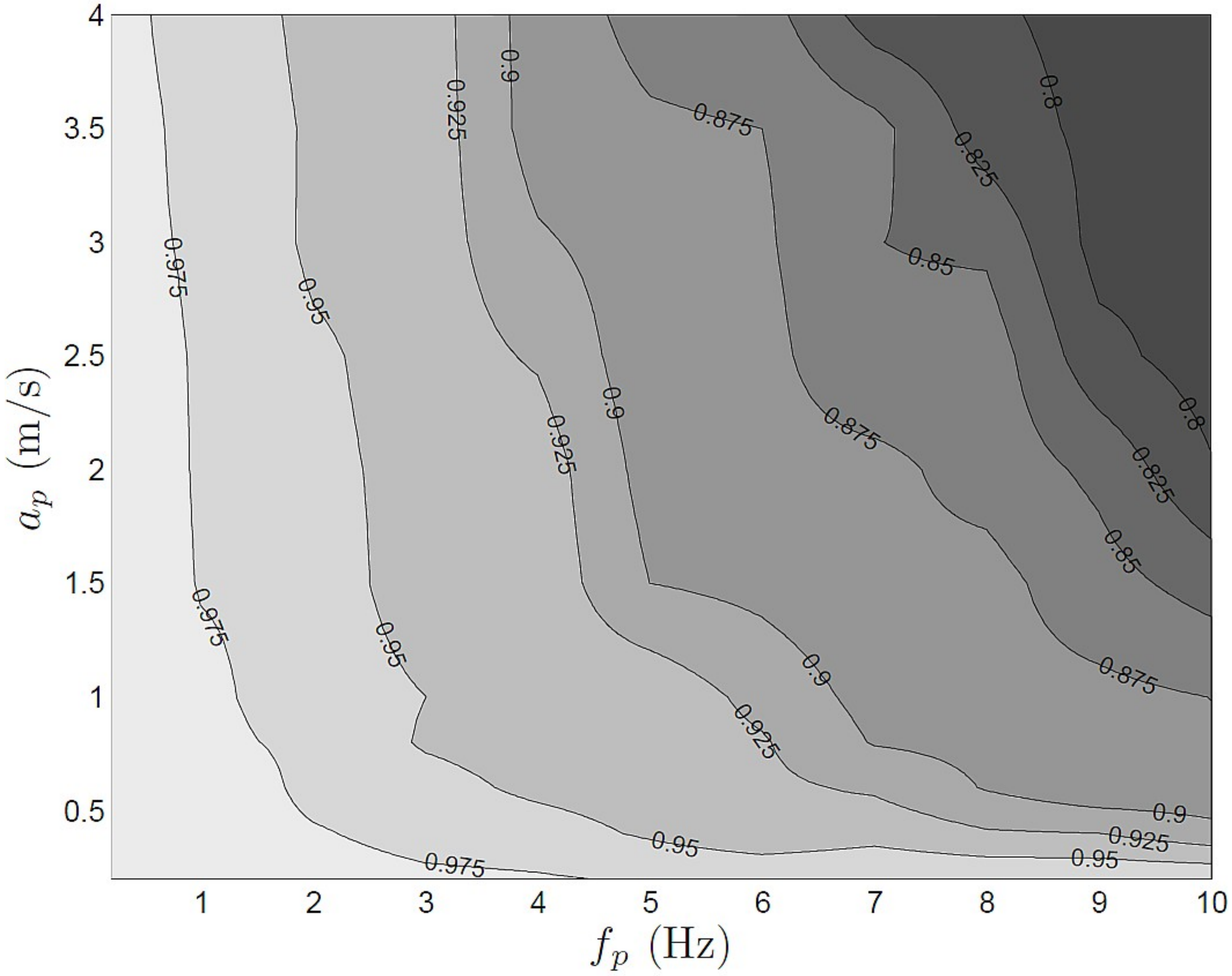
$$\begin{cases} u_r(t) = \sum_{k=1}^{\boxed{k_c}} \mathrm{IMF}_k(t) \longleftrightarrow u_i(t) \\ p_r(t) = u_p(t) - u_r(t) \longleftrightarrow p_i(t) \end{cases}$$

Assessment of the recovering procedure

$$C = \frac{\langle u_i(t)u_r(t)\rangle}{\sqrt{\langle u_i^2\rangle}\sqrt{\langle u_r^2\rangle}}$$

C) Performances of EMD: results

$$C = \frac{\langle u_i(t) u_r(t) \rangle}{\sqrt{\langle u_i^2 \rangle}\sqrt{\langle u_r^2 \rangle}}$$



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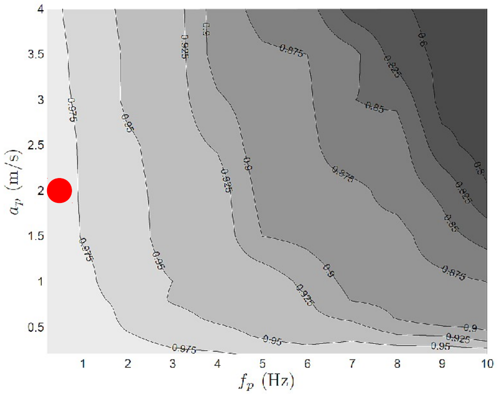

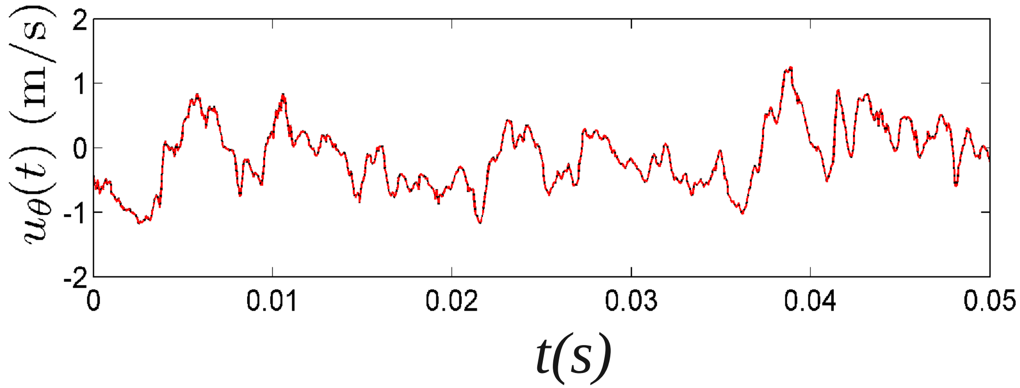

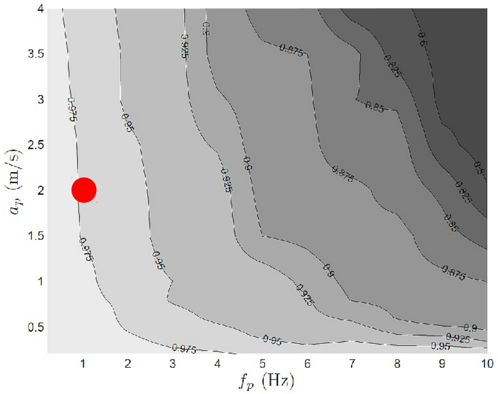

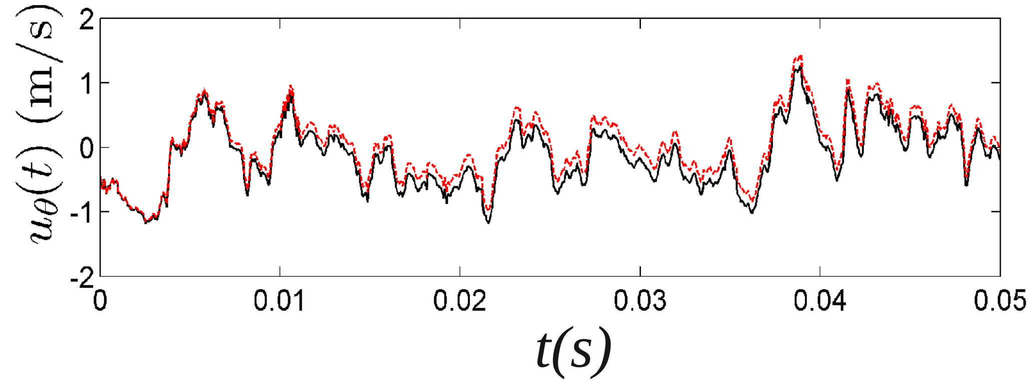

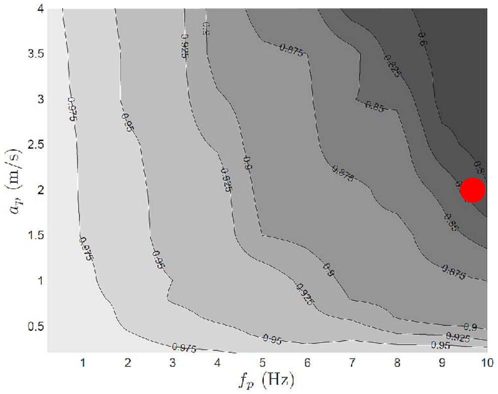

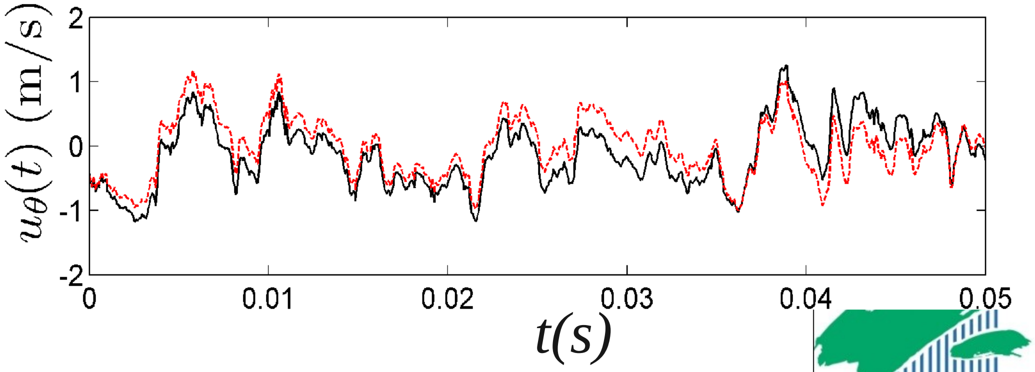

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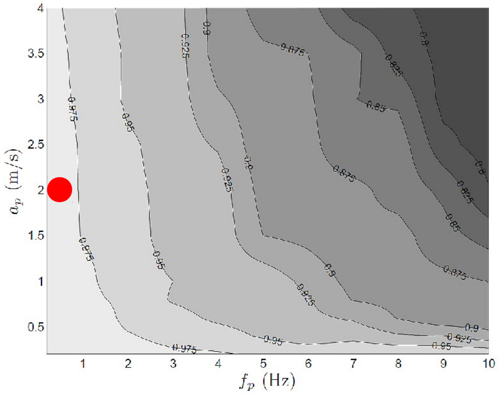
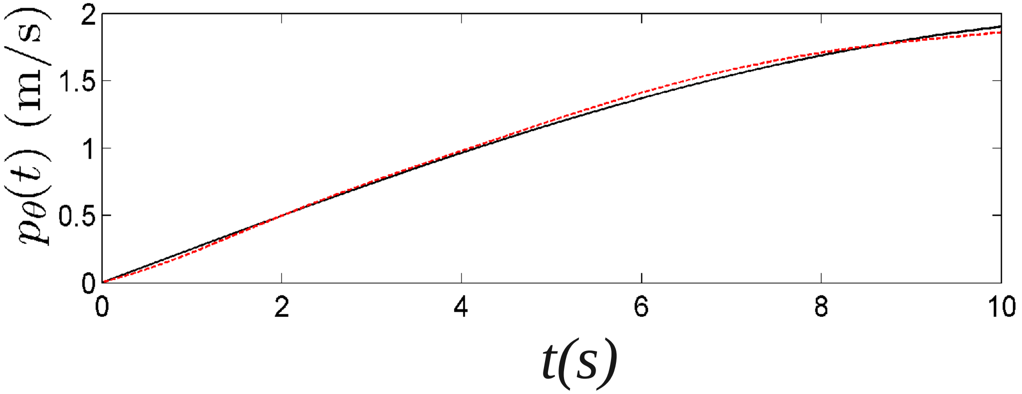
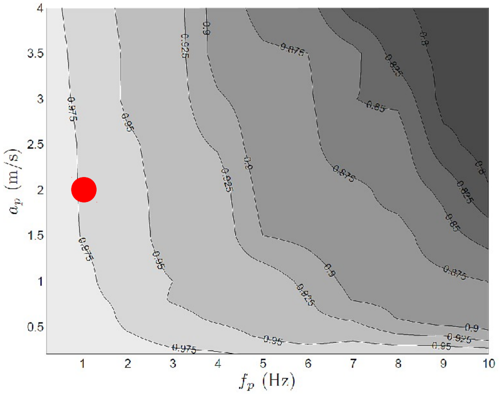
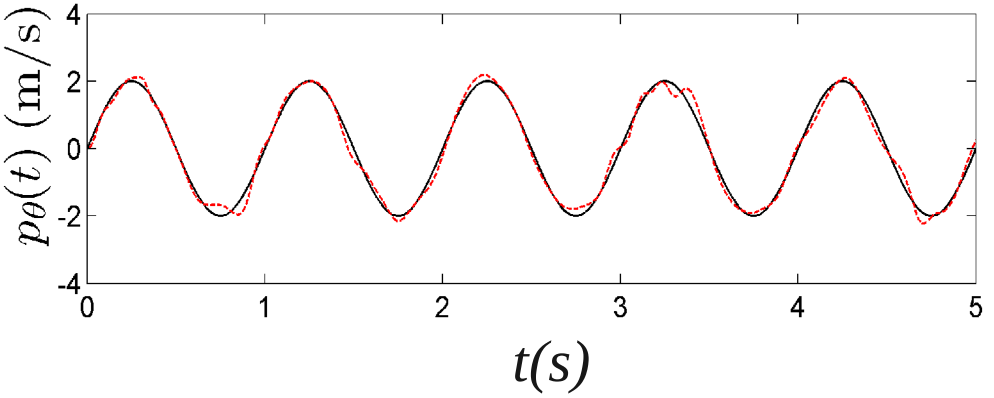
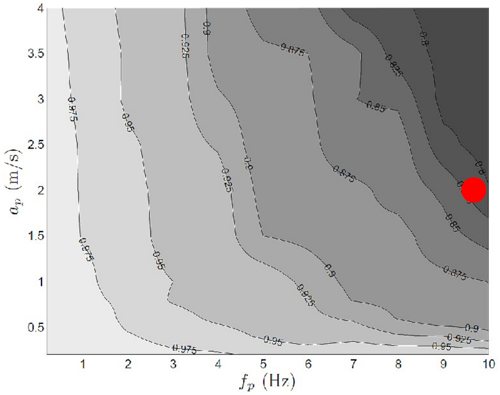
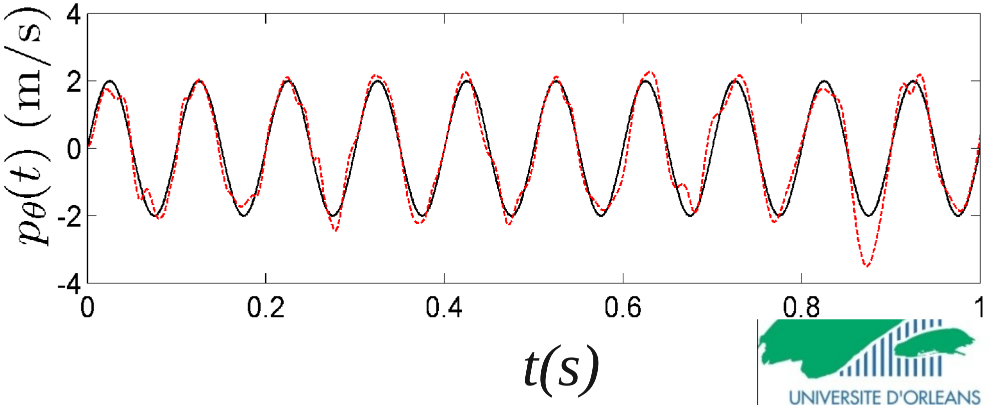










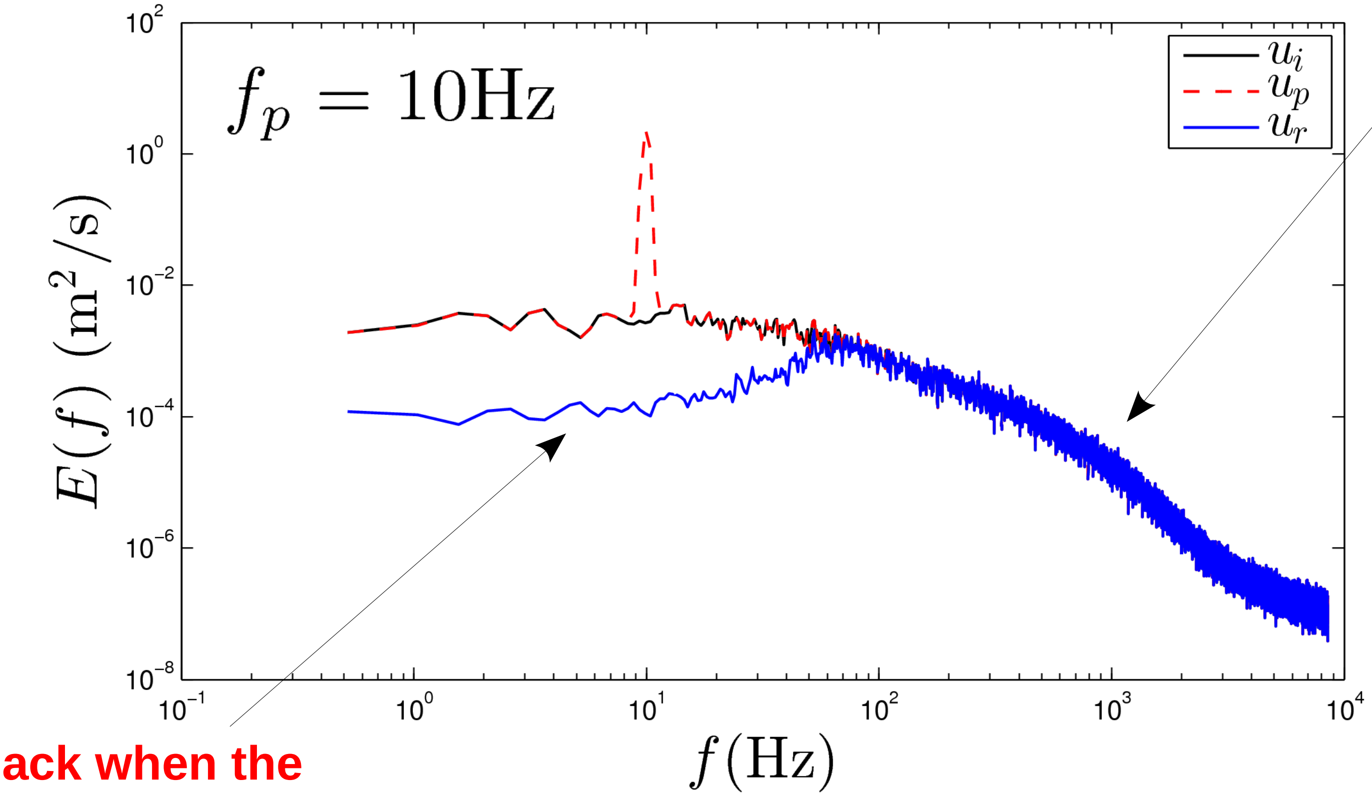


C) Performances of EMD: mode mixing

Energy spectrum: $\langle u_\theta^2 \rangle = \int_0^\infty E_\theta(f)df$

Small and inertial range successfully recovered

« high-pass » filtering



Drawback when the frequency range overlaps

Mode mixing

Large-scale statistics are badly estimated

C) Performances of EMD: mode mixing

Solutions to prevent or tamper with “mode mixing”:

- ➡ Extended-EMD (noise addition)
  (Wu and Huang, 2009)
- ➡ Masking signal (appropriate tone addition)
  (Deering and Kaiser, 2005)
- ➡ Modulated-Demodulated EMD (amplitude modulation)
  (Segalen et al., 2011)

C) Performances of EMD: mode mixing

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Modulation step

$$h(t) = s(t)\cos\left(2\pi f_0 t\right) + \mathcal{H}\left[s(t)\right]\sin\left(2\pi f_0 t\right)$$

Carrier frequency

Hilbert transform

EMD step

Demodulation step

$$\mathrm{IMF}_s(t) = \mathrm{IMF}_h(t)\cos\left(2\pi f_0 t\right) - \mathcal{H}\left[\mathrm{IMF}_h(t)\right]\sin\left(2\pi f_0 t\right)$$

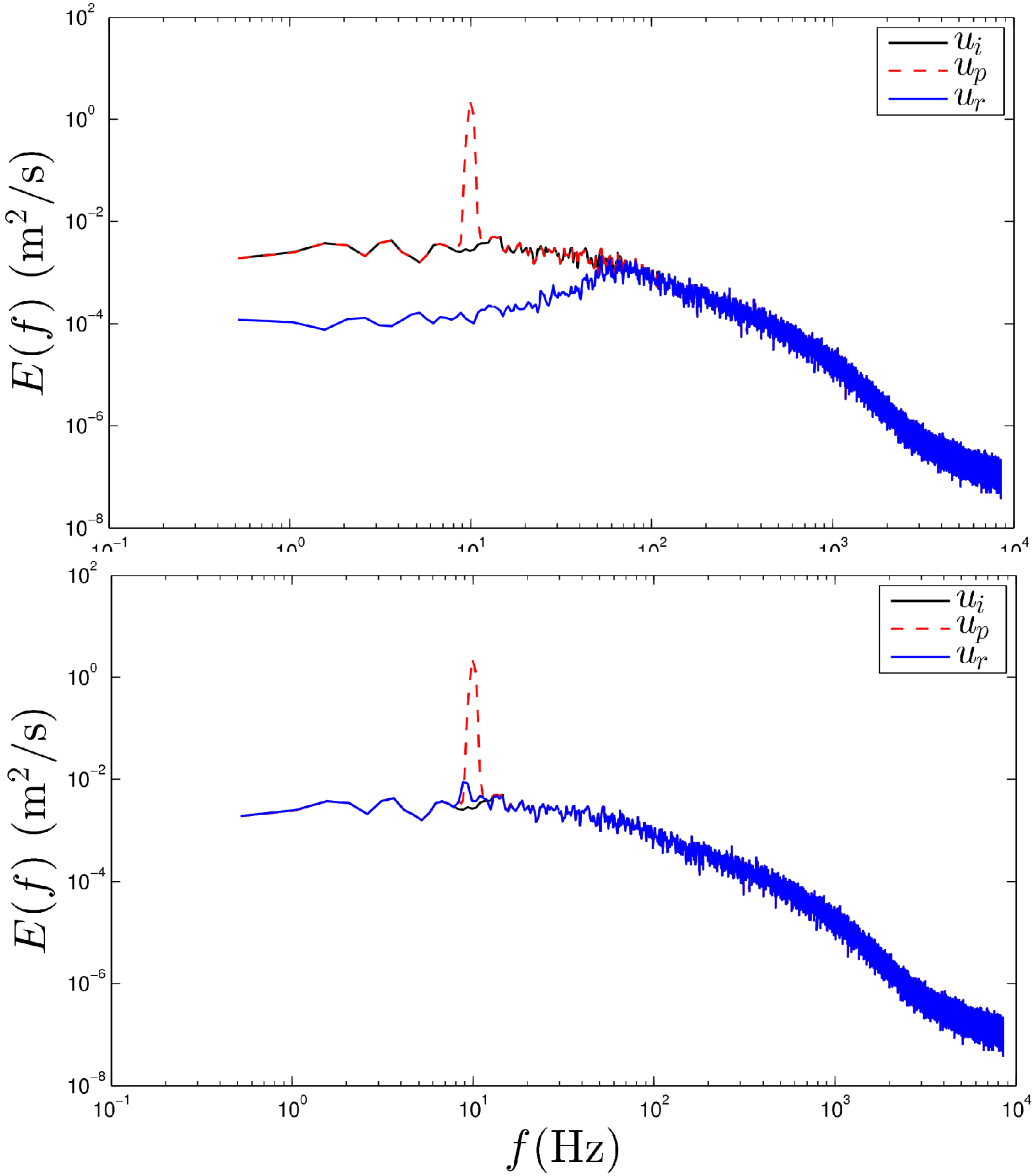
C) Performances of EMD: mode mixing

Standard EMD

MD-EMD

$$f_p = 10\text{Hz}$$

$$f_0 = 9\text{Hz}$$



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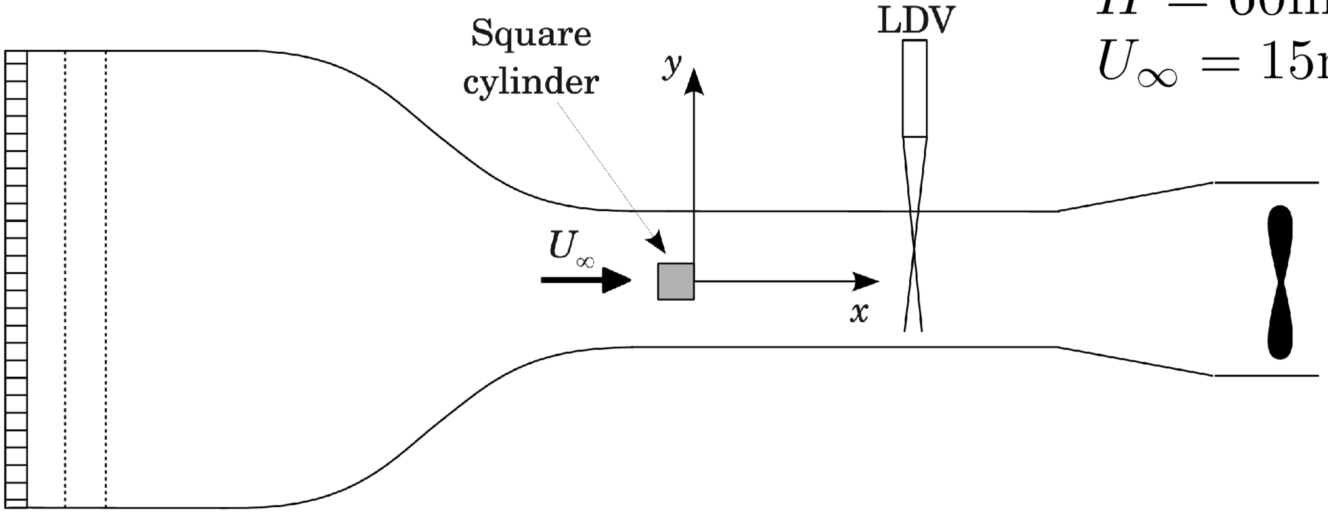
D) Application to turbulent wake: set-up

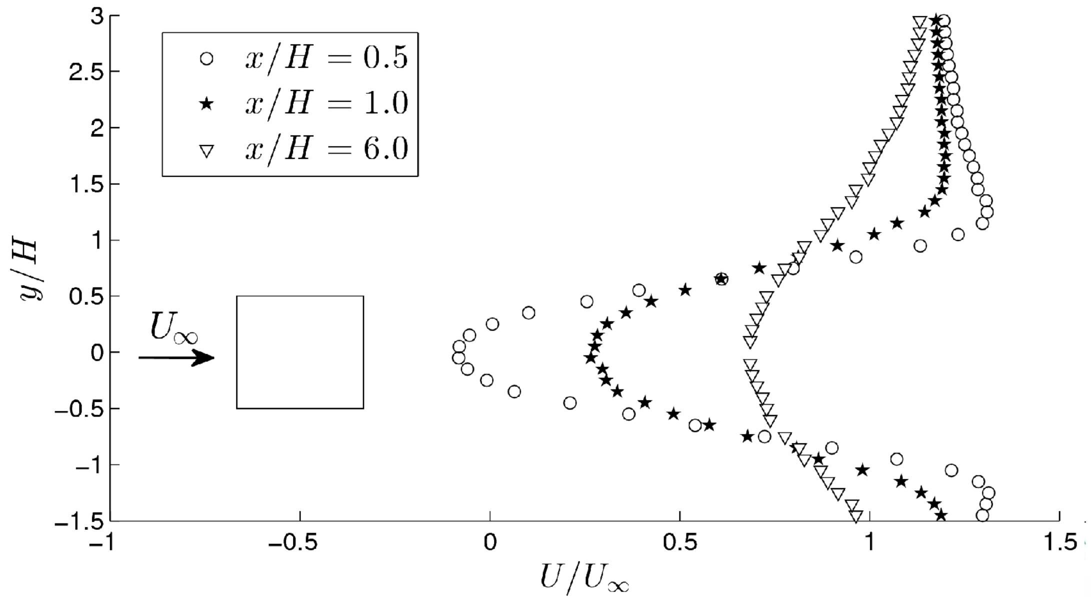


$$\left.\begin{aligned} H &= 60\text{mm} \\ U_\infty &= 15\text{m/s} \end{aligned}\right\} Re = 60000$$

$$St = \frac{fH}{U_\infty} = 0.145$$

LDA 2 components
- ➢ Streamwise (u)
- ➢ Transverse (v)

Near-wake → x/H = 1



D) Application to turbulent wake: algorithm

LDA measurements

$u(t) - U$

Modulation step

$u_{mod}(t)$

EMD step

Ressemblance criterion

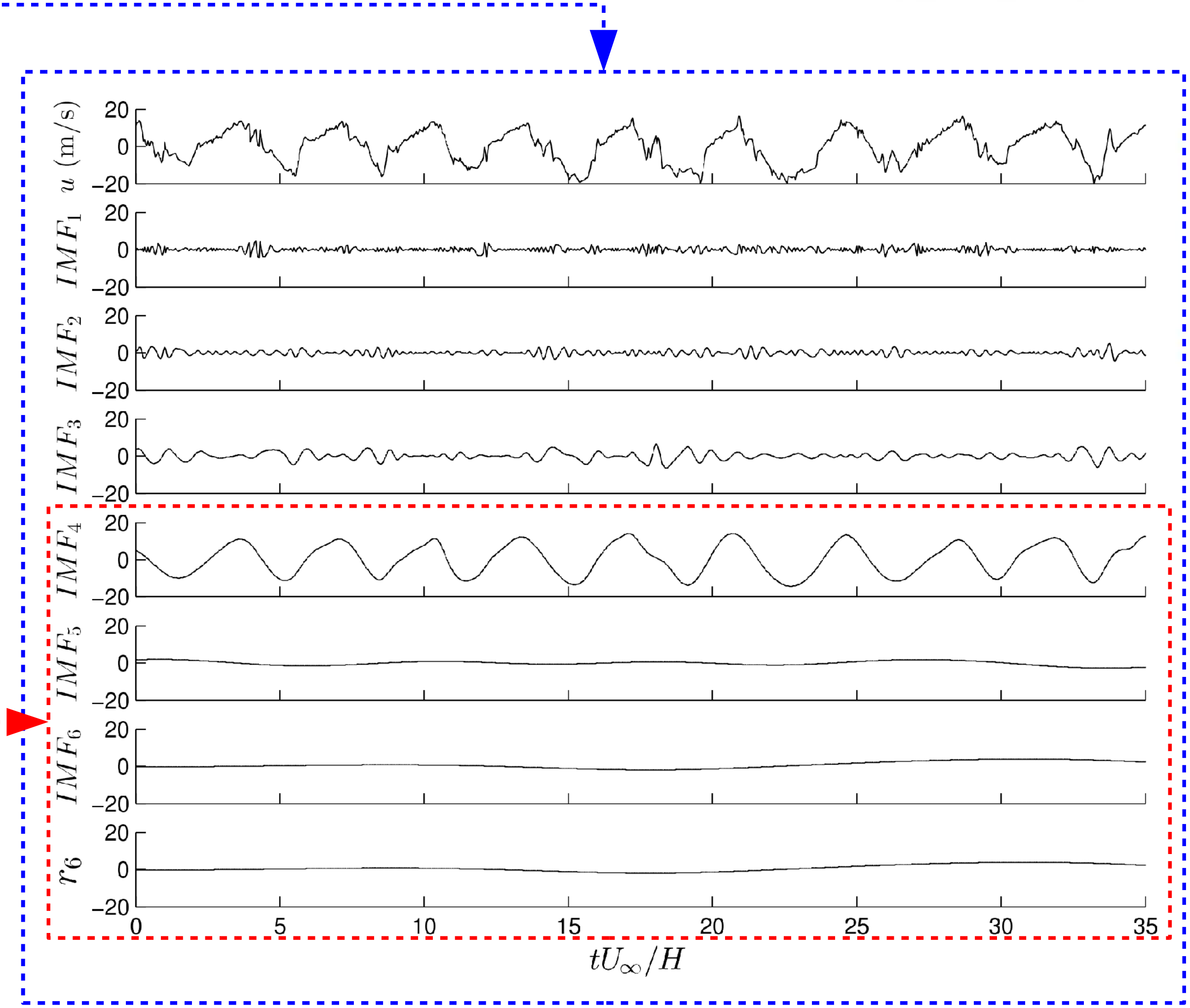
$\tilde{u}_{mod}(t)$

Demodulation step

$\tilde{u}(t)$

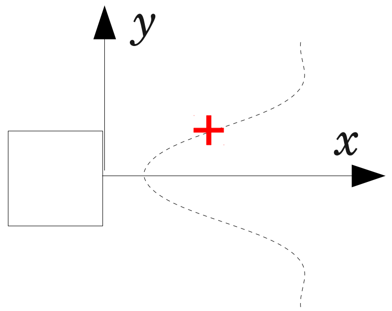
Substraction step

$u'(t)$

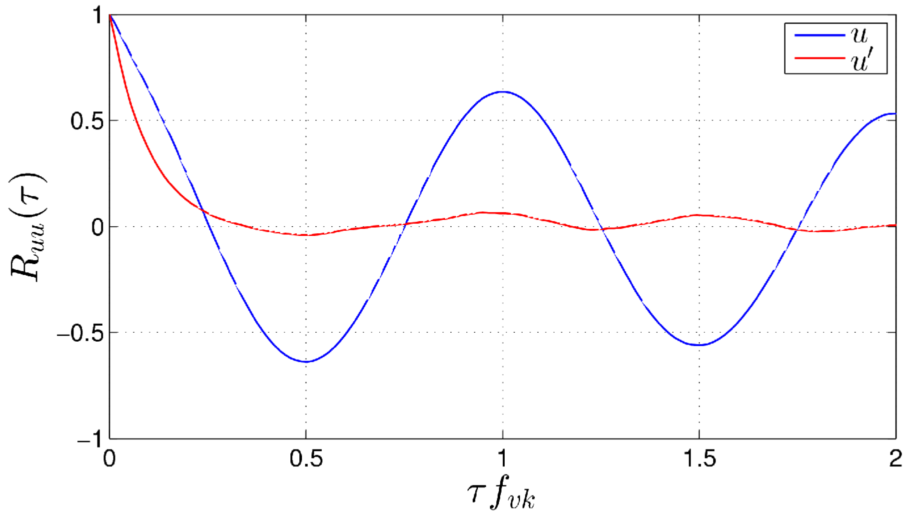


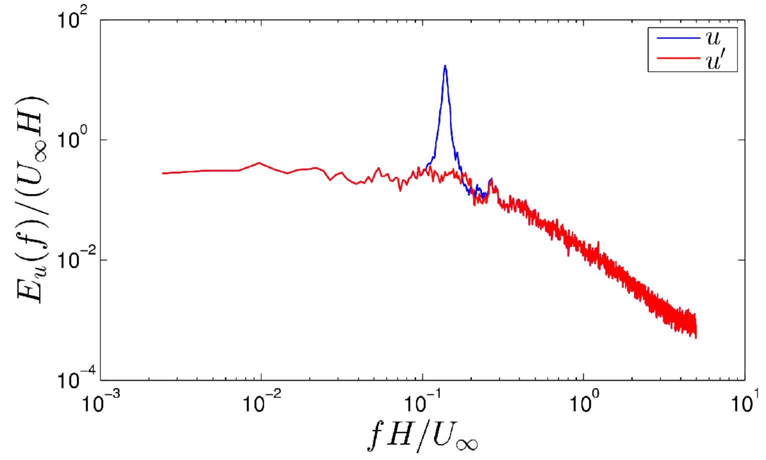
$$u(t) = U + \tilde{u}(t) + u'(t)$$

D) Application to turbulent wake: results



$$R_{uu}(\tau) = \frac{\langle u(t+\tau)u(t)\rangle}{\langle u(t)^2\rangle}$$

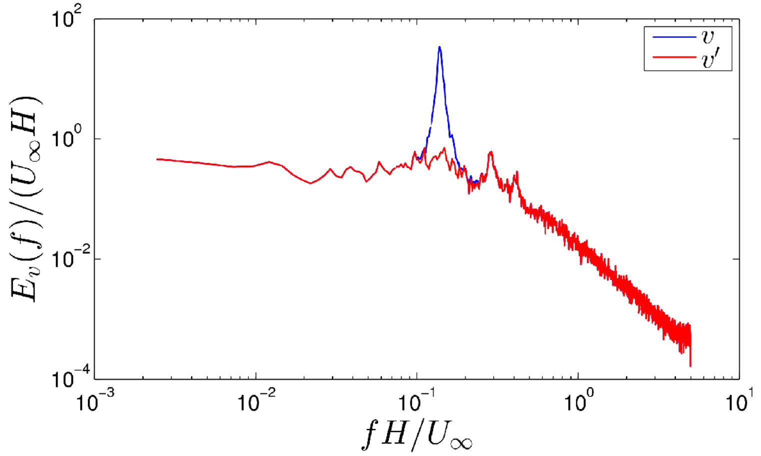




$$T_{u'} = \int_0^\infty R_{u'u'}(\tau)d\tau$$

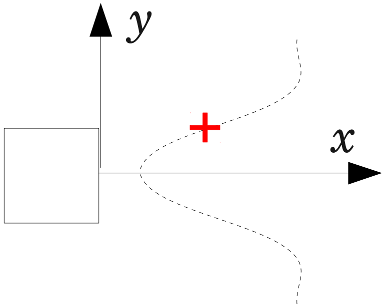
$$\boxed{T_{u'} \approx T_{v'} \approx 2.5\text{ms}}$$

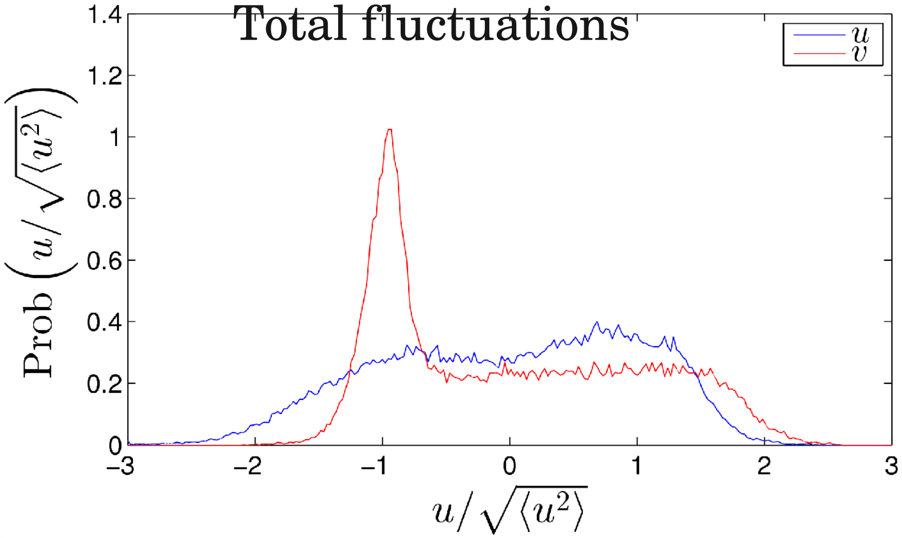
$$f_{u'} \approx 400\text{Hz} \gg f_{vk} \approx 35\text{Hz}$$

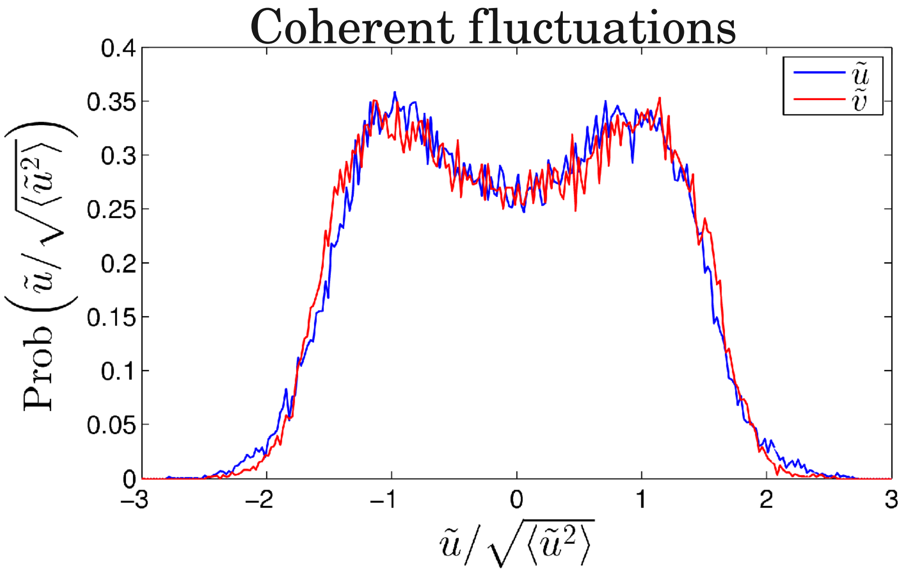


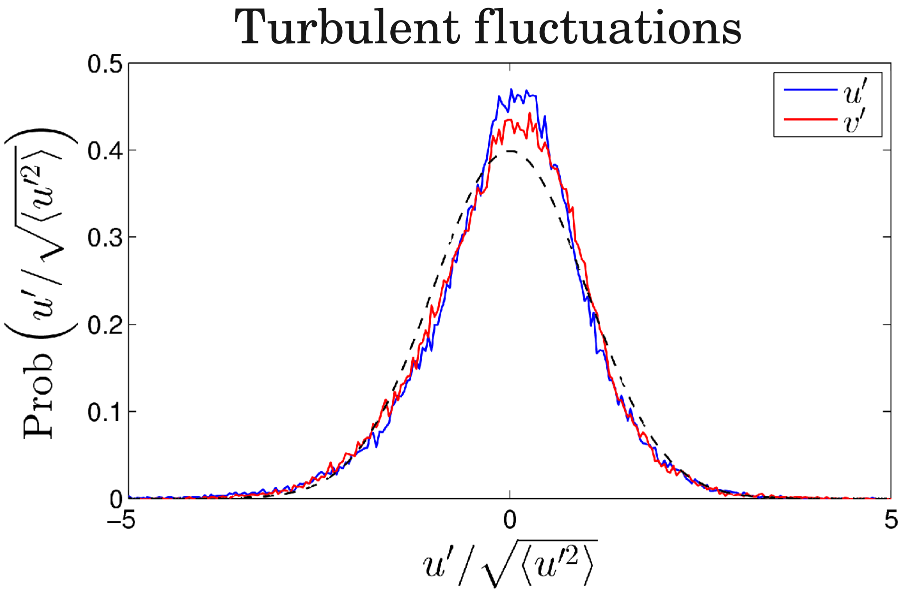
$$\boxed{\Rightarrow \frac{f_{u'}H}{U_\infty} \approx 1.6}$$

D) Application to turbulent wake: results



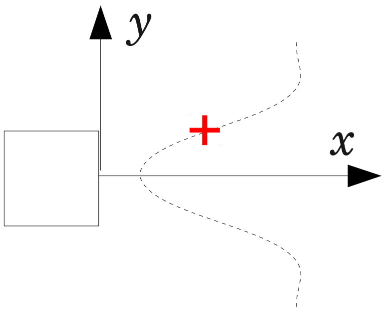




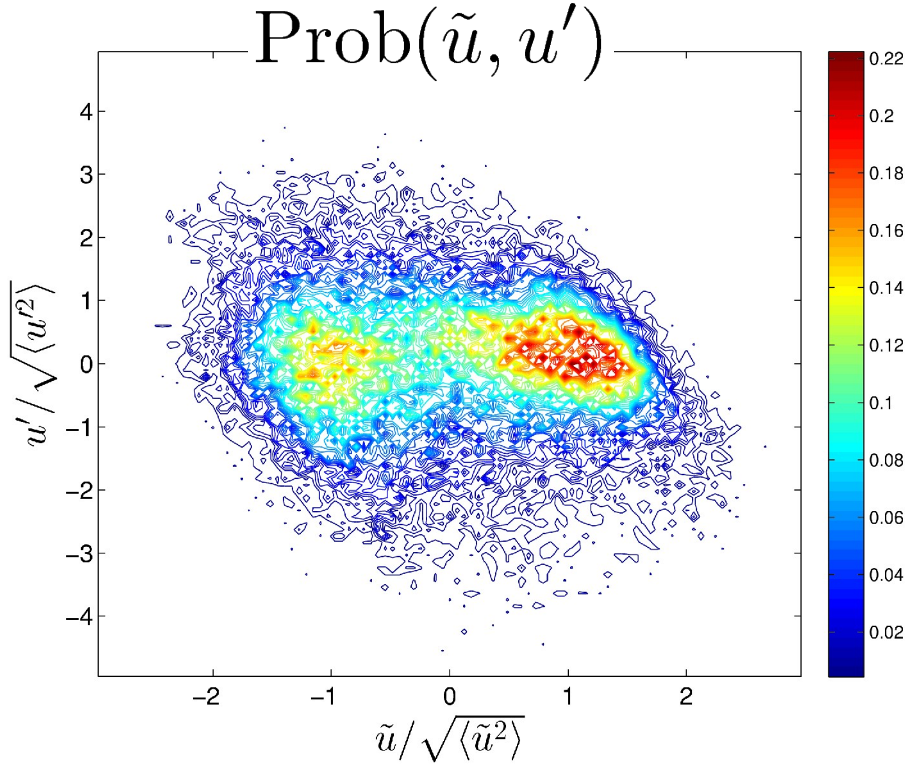


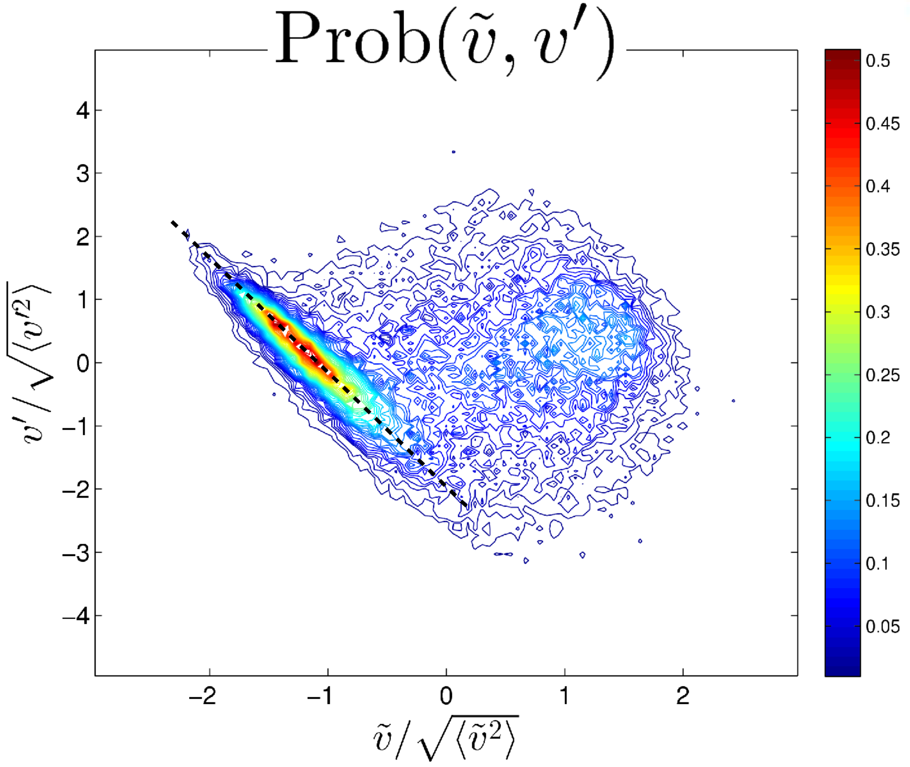
⇒ Statistical dependencies?

D) Application to turbulent wake: results

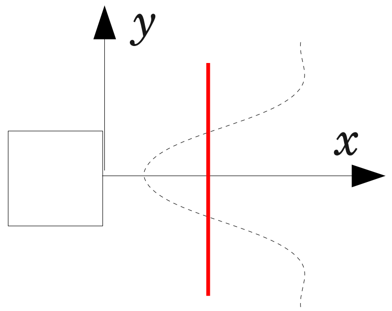


Statistical dependency → joint-PDF

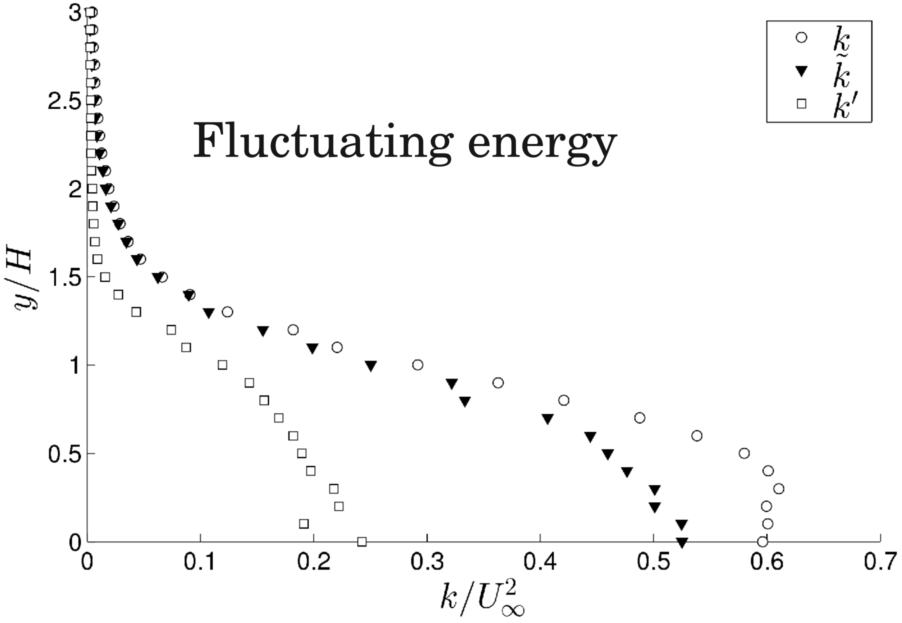

$\mathrm{Prob}(\tilde{u}, u')$

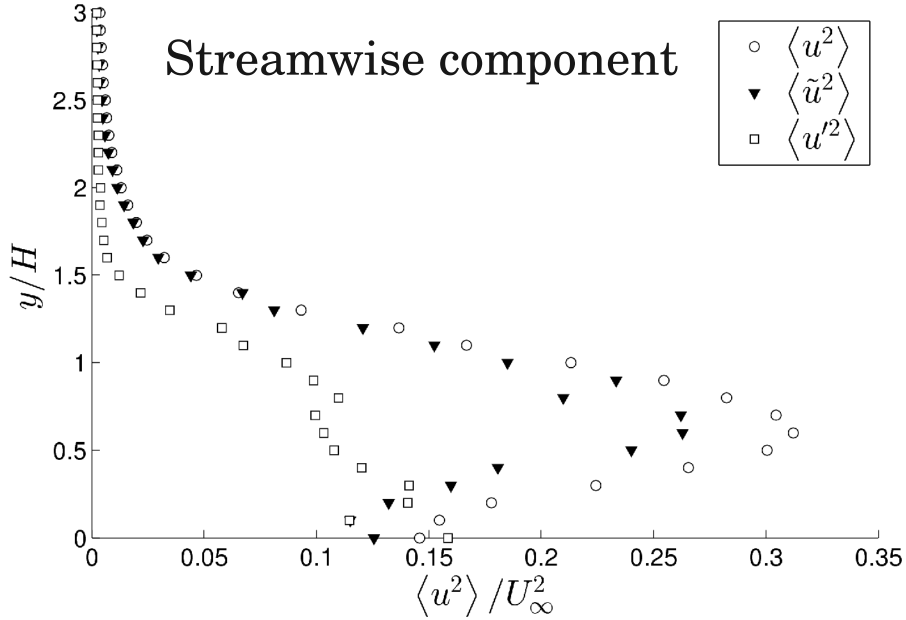

$\mathrm{Prob}(\tilde{v}, v')$

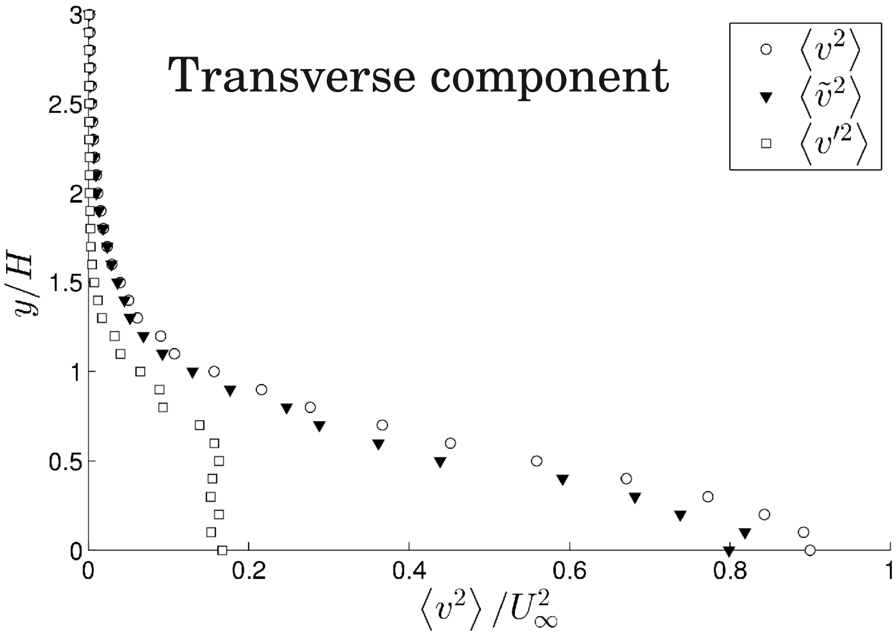
Strong statistical dependency between $\tilde{v}$ and v'



$$k = \frac{1}{2}\left(2\left\langle u^2\right\rangle + \left\langle v^2\right\rangle\right)$$

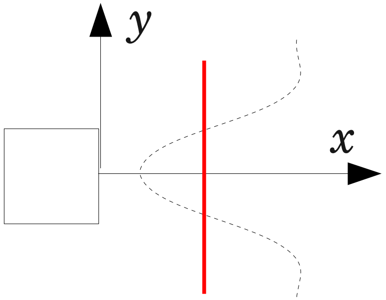

Fluctuating energy


Streamwise component


Transverse component

$$\boxed{\left\langle \tilde{u}^2\right\rangle \le \left\langle \tilde{v}^2\right\rangle \text{ and } \left\langle u'^2\right\rangle \approx \left\langle v'^2\right\rangle}$$

D) Application to turbulent wake: results



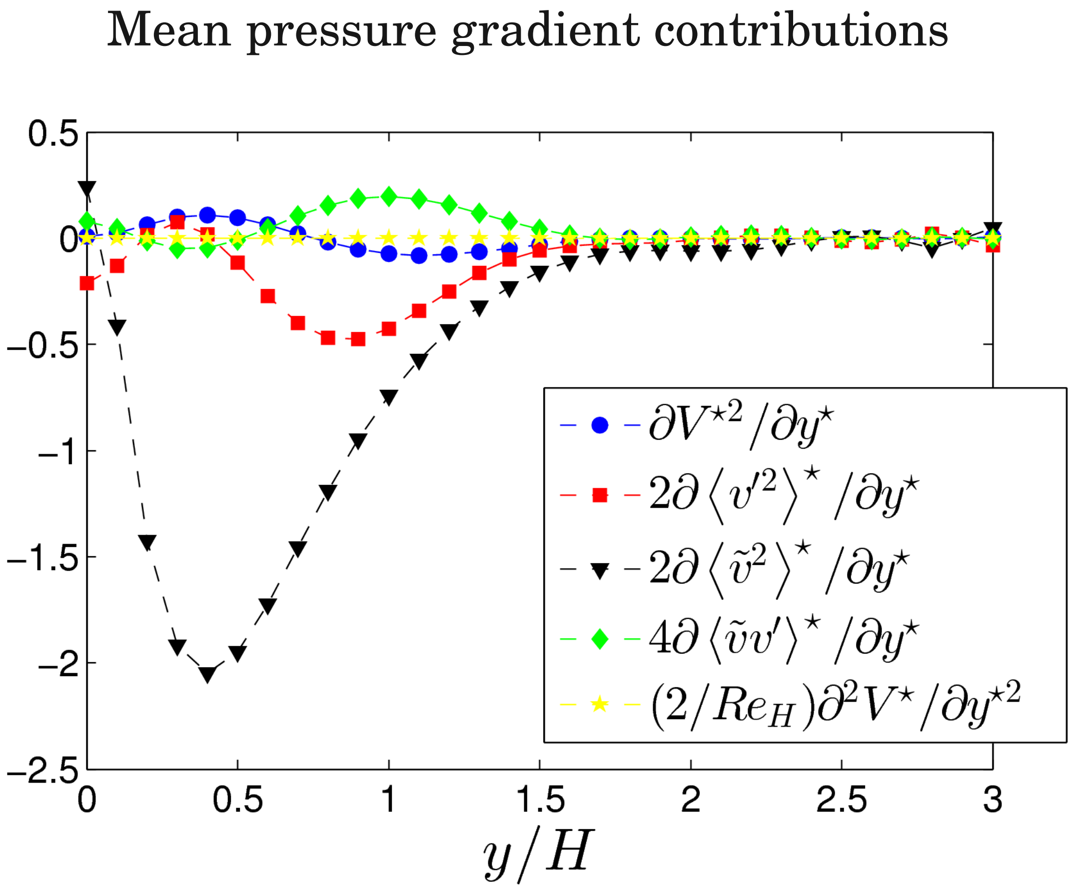
Navier – Stokes

+

Triple decomposition

+

Average operator


Mean pressure gradient contributions

$$-\frac{\partial C_p}{\partial y^\star} \approx \frac{\partial V^{\star 2}}{\partial y^\star} + \overbrace{2\frac{\partial\left\langle v'^2\right\rangle^\star}{\partial y^\star}}^{\text{Dominant term}} + \underbrace{2\frac{\partial\left\langle \tilde{v}^2\right\rangle^\star}{\partial y^\star} + 4\frac{\partial\left\langle \tilde{v}v'\right\rangle^\star}{\partial y^\star}}_{\text{Opposite sign}} - \cancel{\frac{2}{Re}\frac{\partial^2 V^{\star 2}}{\partial y^{\star 2}}}$$

Conclusions

<u>Empirical Mode Decomposition</u> : original method designed to investigate non-linear and non-stationary physics

Fully developed turbulence (homogeneous and isotropic)
+
Low frequency numerical flapping

Separation between both contributions → **« resemblance » criterion**

Mode mixing problem “resolved” using MD-EMD

Analysis of the velocity fluctuations in the near-wake square cylinder
Triple decomposition (separation between coherent and turbulent contributions)

Turbulence frequency range >> VK instability (comparison with KH?)

Strong statistical dependency between the transverse components (entrainment?)

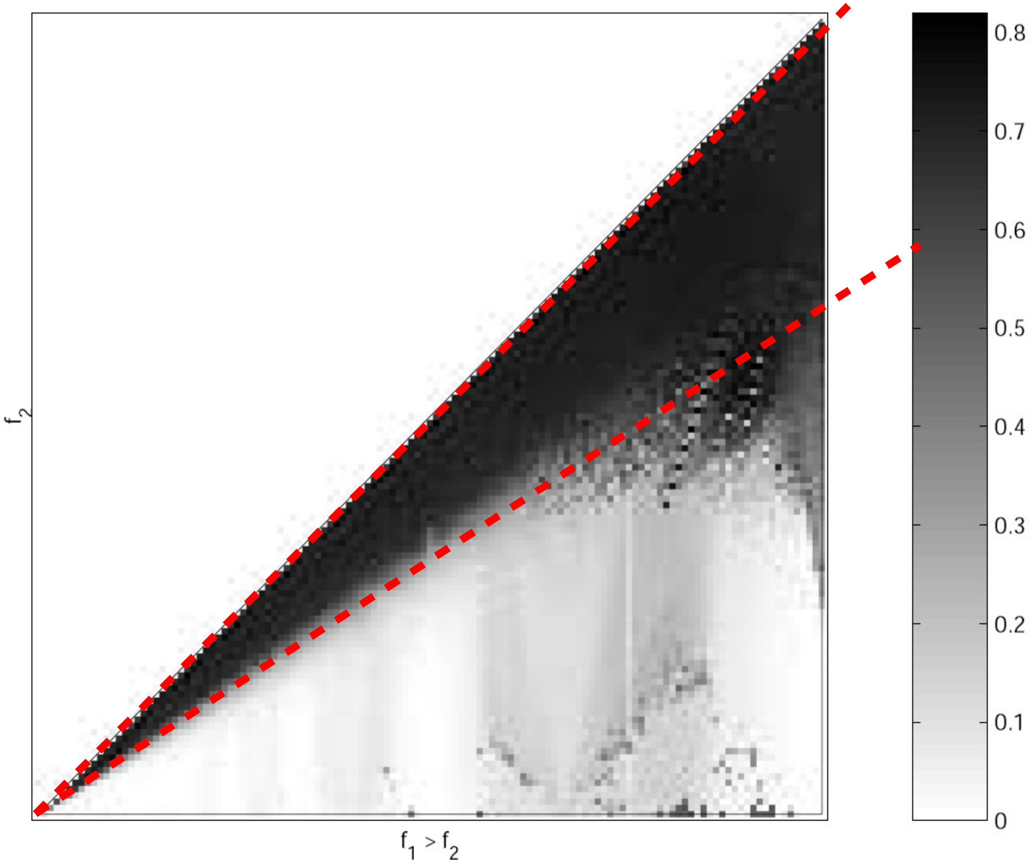
Pressure gradient (somehow force) mostly dominated by coherent contribution

Offres d'emploi à PRISME :

➡ **Bourse de thèse “contrôle adaptatif de décollement sur une rampe”**
Compétences recherchées : méca flu expérimentale
Labex CAPRYSSES
Disponible

➡ **Financement Poct-doc “Interaction flamme-vortex” (12 mois)**
Compétences recherchées : méca flu expérimentale
ANR IDYLLE (DGA)
démarrage Mars 2014

$$s(t) = \cos\left(2\pi f_1 t\right) + \cos\left(2\pi f_2 t\right)$$



“confusion” error *(Rilling et al., 2003)*

$$\boxed{\frac{f_2 - f_0}{f_1 - f_0} \leq \frac{f_2}{f_1}}$$

Amplitude Modulation $h(t) = \cos\left(2\pi(f_1 - f_0)t\right) + \cos\left(2\pi(f_2 - f_0)t\right)$